<u>Comments Regarding the Proposed Revisions to</u>
<u>19 TAC Chapter 111.</u>
<u>Texas Essential Knowledge and Skills for Mathematics</u>
<u>Grade-by-Grade Review</u>

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General Comments:

1. For K-8, standards in every grade should be critically reviewed with an eye towards merging standards that essentially cover different aspects of the same mathematical idea. Unfortunately, within the TEKS, in many cases three separate standards exist that expect students to (a) generate (b) model, and (c) solve particular mathematical content. Clearly, this is tedious and ultimately subtracts from the readability of the standards.
   While some suggestions to that effect are captured in the Grade-by-Grade Review, many less-prominent cases are left unmentioned and this is detrimental to the overall standard.

2. The proliferation of the standard mentioned in the previous item is not merely a stylistic issue. Test-makers need to assess every standard, and key standards often need multiple test items. Proliferation of standards directly drives-up the number of test items, making testing more expensive to develop and more time consuming to administer.

3. The syntax of the standards, particularly in K-8, needs careful review. There are many more cases than enumerated in the following pages that, as with any important document, require careful copy-editing.

4. In many cases, the standards are written in an opaque style, describing unnamed "relationships" between mathematical objects. The clarity of the standards will be greatly enhanced by explicitly listing the specific relationships that students are expected to know.

5. Neither the current nor proposed standards reflect the accepted importance of achieving fluency in arithmetic using the standard algorithms by the end of the elementary grades.

6. In some cases, the standards are not listed in a coherent order within each grade. Clearly, reordering of the content should be an integral part of final editing.

7. The following Grade-by-Grade Review includes neither Data Analysis nor Personal Financial Literacy. I have addressed those two categories here.

a. Data Analysis:

Generally speaking, the Data Analysis strand seems well done. It is defined as a minor strand in K-5 and, as such, it does not overly intrude on the main mathematical content in K-5. Its content progression has been done reasonably well, and its readability is much better than that of the main content.

b. Personal Finance Literacy:

The case of financial literacy is different from the Data Analysis one. While the mathematics involved are quite elementary, the context in which it needs to be applied seems challenging to young students. Human capital is a rather complex notion, yet first graders are already expected to understand its relation to work even though the precise relationship between human capital and work is known to few adults. Does human capital promote employment? We would like to think so, yet this relationship seems to not always hold true in times of economic crisis. Does increased human capital promote economic activity? We believe so, but we also have the case of Japan. Second graders are already supposed to distinguish between responsible and irresponsible borrowing, yet we all know that this is not an easy question to answer, and it greatly depends on individual judgment. The list of expectations quickly grows and gets even more complex across the grades, yet mathematics remains only a small fraction of the knowledge involved in the answers.

It seems unclear, at this point, if inclusion of this strand in the mathematics curriculum will prove beneficial. The issues it raises are non-mathematical and complex, and we remain uncertain that elementary teachers will be able to do them justice. It is also unclear whether middle school mathematics teachers are the optimal ones to deliver this content at the middle grades.

What is clear is that this strand will detract from the focus and coherence of the mathematics curriculum. It perhaps could be handled as one or more focused units delivered during a middle school year when student maturity, interest, and understanding of such issues are growing. Another possibility is simply to experiment with it in a small number of districts and monitor its success.

Kindergarten

Notes:

- In the following text the red font indicates direct quotes from the draft TEKS.
- In the following text all numerical references are to the Knowledge and Skills standards

1. Standard (2) works with numbers up to 20. This is a positive change as compared to the Common Core (CC) Standards, which unreasonably push students to work with numbers up to 100 at this level.
2. (2)(C): count a set of objects up to at least 20 and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement. This is language was taken from CC, yet the original had "regardless of their arrangement **<u>or order</u>**" and the "or order" was dropped here. Recognizing that order of counting does not matter is more important than objects' arrangement and the original wording should be restored.
3. (2)(D): recognize instantly the quantity of a small group of objects in organized and random arrangements.

   This standard addresses subitizing, the ability to recognize the size of a small set at a glance and without counting. This ability is developed when children repeatedly see the same number of objects organized in the same pattern. A good example is the patterns on the faces of a die, although any fixed pattern would do. So, for example, there is no particular advantage to show 3 as a series of 3 diagonal dots as on a die, or in a triangular arrangement, but the key is to show them always in the same way. And therein lies the first misunderstanding of this standard: it should have never referred to random arrangements. In fact, using random arrangements as the standard directly undermines the development this skill.

   Moreover, this skill is a scaffolding skill rather than a goal in itself. It simply saves children time of counting objects when they deal with manipulatives at an early age. There is no research supporting the proposition that the skill is, in and of itself, valuable to developing mathematical numeracy. Adding it as an explicit learning objective is wrongheaded. At best, it is a pedagogical measure rather than a mathematical objective. The standard should be eliminated.
4. Standard (2) includes 9 sub-standards, many of them dealing with the same content from a slightly different angle. This adds to the verbosity and unwieldiness of the standards. A much better approach is to merge many of them together, which will increase their focus and clarity of the document. For example the following ones could be easily merged:

- generate a set using concrete and pictorial models that represents a number that is more than, less than, and equal to a given number up to 20;
- generate a number that is one more than or one less than another number up to at least 20;
- compare sets of objects up to at least 20 in each set using comparative language;
- use comparative language to describe two numbers up to 20 presented as written numerals;

A suggested merged language is:

- *generate and use language to compare numerals and sets of 20 or fewer objects including sets with one less or more objects than others*

5. Within the proposed changes, the proliferation of minute standards creates some that are surprisingly alike and consideration should be given to eliminating such complete, or nearly complete, duplications. For example:

- (3)(C): explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.
- (5)(B): represent addition and subtraction with objects, drawings, situations, verbal explanations, or number sentences.

There seem to be little difference between explaining and representing using "drawings, situations, and verbal explanations."

6. Some geometry standards set forth in the proposed changes seem to be ill advised. For example, asking for the use of "formal geometric language" when describing attributes of shapes (standard (6)(D)) may be premature. First, I assume that the intent was to describe them simply using "geometric terms"–I am unaware of such a thing as a "formal geometric language." More to the point, it is questionable whether insisting on formal terminology in early grades is actually conducive to development of mathematical and geometrical understanding. There is evidence that it sometimes results in teachers focusing more on developing formal vocabulary instead of fostering the understanding of concepts themselves. It is recommended easing off on requiring formalism in K-2.

7. Standard ((6)(E)) requires distinguishing between regular and irregular shapes. This requires a depth of understanding that seems unnecessary and beyond what one can reasonably expect in Kindergarten. The standard should be eliminated.

Grade 1

1. In this grade, students deal with numbers up to 120. This seems strange and unnecessary–100 would be a much more natural limit, as is common in many states. It could be that the 120 was lifted from the Common Core, where it is–in my opinion–also a mistake. What seems to have happened is that the Common Core made Kindergarten students work with numbers up to 100, even higher than places like Singapore, Japan, or Korea who expect their students to work up to 100 only in the grade one. Consequently, the Common Core had a dilemma with grade one–1,000 seemed too high, while 100 was already handled in kindergarten, so it settled on the weird 120. Texas correctly set the Kindergarten expectation to 20 and there is no reason to set grade one above 100. Two-digit addition and subtraction is what grade one deals with, and 100 should be set as the natural limit.
2. (2)(A): recognize instantly the quantity of structured arrangements such as seen on a die or a ten frame;

   This one is, yet again, a subitizing-related standard similar to item #2 in Kindergarten. For similar reasons, it does not belong here and should be removed.
3. Here is another example of unnecessary proliferation of very narrow standards. The two standards below can be easily merged, as can many others.
   - (5)(B): skip count by twos, fives, and tens to 100;
   - (5)(C): skip count by twos, fives, and tens to determine the total number of objects up to 120 in a set;
4. (5)(D): use relationships to determine the number that is 10 more and 10 less than a given number up to 120;

   I am not sure what kind of relationships the authors have in mind. The standard makes no sense as written.
5. (5)(G): determine the unknown whole number in an addition or subtraction equation when the unknown may be any one of the three or four terms in the equation;

   This is another good example of overcomplicated and clumsy language. I could perhaps guess what the "three terms" means, but I can interpret "four terms" in multiple ways. A shorter description and some examples would go a long way to clarify the meaning.
6. (7)(B): demonstrate that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other;

   I am unsure how one can "demonstrate" that this is true, unless students know how to use rulers or similar but those show up only in later grades. Perhaps it was meant to be "understand" or "know"? Seems like a poor paraphrasing of the CC standard 1.MD.2: “*understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps*.”

7. Like in Kindergarten, this grade’s standards overstress the need for using formal, geometrical terms and repeatedly use the meaningless phrase “formal geometric language.” (6)(D), (6)(E).

Grade 2

1. Grade two seems to follow the previous grade in expecting students to work with numbers up to 1,200. Seems like 1,000 would be much more natural limit.
2. More examples of very narrow standards that can be trivially merged together to avoid unwieldy proliferation:
    - (2)(A) and (2)(B)
    - (2)(E) and (2)(F)
    - (2)(D) and (2)(G)
3. (2)(E): locate the position of a given whole number on an open number line;
   It makes no sense to "locate" a number on an open (i.e., empty, without markings) number line.
4. Confused and hard to understand language:
   (4)(B): use mental strategies, flexible methods, and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers;
   I am not sure what algorithms based on the knowledge of place value are, nor what are algorithms based on equality, nor which flexible methods are intended here. Perhaps what this simply meant to say was:
   "use mental strategies and algorithms based on place value to add and subtract two-digit numbers;"
   Similarly, standard (4)(C): solve one-step and multi-step word problems involving addition and subtraction of two-digit numbers using a variety of strategies based on place value, including algorithms;
   is unclear whether it is about problem solving or about computation. If it is about the former then the whole last clause starting with "using variety …" is irrelevant. If it is about computation, how is it different from (4)(B) above?
5. Standard 7 uses the term "relationship" in strange ways.
   (7)(A): use relationships and objects to determine whether a number up to 40 is even or odd;
   It is unclear which "relationships" the author has in mind here.
   (7)(B): use relationships to determine the number that is 10 or 100 more or less than a given number up to 1,200;
   In this case, it seems that the author had in mind "understanding of place value" instead of some undefined "relationship."
6. Standard (9)(B) seems identical to the previous grade's standard (7)(C) despite different wording. Perhaps the standard in grade one should be eliminated.
    - Gr. 1: (7)(C): measure the same object/distance with units of two different lengths and describe how and why the measurements differ;

- Gr.2: (9)(B): describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object such as the longer the unit, the fewer needed and the shorter the unit, the more needed;

7. (9)(G): read and write time to the nearest five- and one-minute increments using analog and digital clocks and distinguish between a.m. and p.m.;
   If one can tell time to a one minute increment it makes little sense to mention also five minute increments. It is suggested to drop the reference to the five-minute increments.

Grade 3

1. (2)(A): compose and decompose numbers up to 100,000 in more than one way as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate;
   I am unsure why there should be more than one standard way to decompose a number such as 4321 into thousands, hundreds, tens and ones.
2. I suggest you merge standards:
   (3)(A) and (3)(B)
   (4)(F) and (4)(J)
3. (3)(D): compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts 1/b;
   This is an incorrect paraphrase of Common Core standard 3.NF.1. Suggested rewrite: *"compose and decompose a fraction a/b as a sum of a parts of size 1/b*;"
4. (3)(F): represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;
   A number line is not what one normally considers as being of the same type as "objects or pictorial models."
5. (4)(A): solve with fluency one-step and multi-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction;
   Presumably this should be "two-step" rather than "multi-step." Compare with standard (5)(A) that explicitly speaks of two-steps in an almost identical situation. These two standards should be merged.
   Further, it is unreasonable to expect "fluency" with problem solving. One can expect fluency with routine procedures, but rarely with supposedly non-routine problems.
6. (4)(H): determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;
   The standard needs to provide guidance as to the maximum size of the group, whether the group is evenly divisible into shares or not, and the maximum permissible number of shares.
7. (6)(A): classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language such as vertex, edge, and face;
   It is almost identical to the second grade standard Gr. 2 (8)(C): classify and sort three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes based on attributes using formal geometric language such as vertex, edge, and face;

Further, it misleadingly speaks of two-dimensional figures while it lists only three-dimensional ones. Finally, the "formal geometric language" expression should be replaced with "geometric terms."

8. (6)(E): decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape.
   This makes no sense. Seems to be a garbled paraphrase of Common Core standard 2.G.3. If that is the case, it is suggested to use the original wording of the Common Core second grade standard here. If something else is intended, it needs to be better clarified.

Grade 4

1. (2)(A): interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;
   Compare the clarity of this standard with the addled equivalent in grade three (below). Standards are not quizzes–they should be clear and explicit like this one.
   Gr.3: (2)(B): describe the mathematical relationships found in the base-10 place value system through the 100,000s place;
2. (3)(D): generate equivalent fractions to create equal numerators or equal denominators to compare two fractions with unequal numerators and unequal denominators and represent the comparison of two fractions using the symbols >, <, or =;
   It is difficult to imagine a more convoluted and obscure description of something as simple as: "*Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators. Use symbols >, =, or <, to record the results*." (Common Core, 4.NF.2)
3. (3)(E): represent and solve addition and subtraction of fractions with equal denominators and referring to the same whole using objects and pictorial models that build to the number line such as strip diagrams and properties of operations;
   The second half of the sentence seems garbled. The first half should not refer to "the same whole." Referral to the same whole makes sense only when referring to area models. It makes no sense when used in context of fractional numbers, number line, or similar.
4. (3)(F): estimate the reasonableness of sums and differences using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole;
   The word "estimate" is misleading here. It should be "evaluate" or "assess." After the word "differences" there should be "of fractions." Finally, the reference to "the same whole" makes no mathematical sense in this context.

5. (3)(H): determine fractional and decimal quantities as being close to 0, 1/2, and 1.
   This standard is mathematically incoherent as "being close" is undefined. Pedagogically, using benchmark fractions, makes sense only for verification of addition or subtraction of fractions.
6. (4)(A): add and subtract whole numbers and decimals to the hundredths place using a variety of methods, including pictorial models, the inverse relationship between operations, concepts of place value, and efficient algorithms;
   Does a calculator qualify as an efficient algorithm? Asking an older friend? Why not simply say what you mean – the standard algorithm? And why still use crutches like pictorial models if the standard algorithm is understood and mastered?
7. (4)(E) : represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;
   Is the dividend divided evenly by the divisor, or are remainders permitted?
8. (4)(G): use strategies, including rounding to the nearest 10, 100, or 1,000 and compatible numbers, to estimate solutions;
   What kind of strategies? Solutions to what?
9. (7)(E): decompose angles such as complementary and supplementary angles into two non-overlapping angles to determine the measure of an unknown angle
   This sentence makes no mathematical sense.
10. (8)(C): solve problems that deal with measurements of length, intervals of time, liquid volumes, masses, and money using addition, subtraction, multiplication, or division as appropriate;
    I doubt many students in grade four know what a mass is. Perhaps "weight" was intended.

Grade 5

1. (3)(B): use strategies and algorithms, including the standard algorithm, to multiply with fluency a three-digit number by a two-digit number;
   Why not simply "multi-digit by multi-digit"? And why not simply "the standard algorithm" instead of confusing the issue with "strategies"? The Common Core expects students to fluently multiply multi-digit whole numbers with standard algorithms in this grade.
2. Standards (3)(D) through (3)(G) should be merged. They all simply refer to multiplication and division of decimals.
3. (4)(A): identify prime and composite numbers using patterns in factor pairs;
   It is unclear how to identify the factor pairs and which "patterns" should be used. Up to how much?
4. (4)(B): represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;
   This seems like a completely unhelpful standard. "Multi-step"? How many steps? What kind of problems? Range of possible numbers?
5. (6)(A): recognize a cube with side length of one unit as a unit cube having one cubic unit of volume and the volume of a three-dimensional figure as the number of unit cubes (n cubic units) needed to fill it with no gaps or overlaps if possible;
   The volume of a 3D figure does not depend on whether it is possible to fill it with unit cubes with no gaps. The standard is only partially coherent.
6. (8)(A): describe the key attributes of the coordinate plane and the process for graphing ordered pairs of numbers in the first quadrant;
   What are the "key attributes" of "the coordinate plane"? Is this a quiz? Why isn't the standard explicit about what it expects?

Grade 6

1. (2)(C), (2)(D) and (2)(E) ought to be merged.
2. (2)(F): extend representations for division to include fraction notation such as a/b represents the same number as a ÷ b where b ≠ 0;
   It is doubtful many students—or many teachers—will understand that the point of this standard is to make sure that students understand that the fraction a/b can be interpreted as a division of "a" by "b."
3. (3)(A): use an area model to represent fraction multiplication and decimal multiplication;
   Using an area model for decimal multiplication was previously covered in standard (3)(D) in grade five. Standards should not repeat themselves across grades.
4. (3)(D): represent integer operations with concrete models and connect the actions to algorithms;
   The meaning of this standard is obscure to me and I suspect many teachers will feel the same. Which actions are supposed to be connected to which algorithms? Good standards should be clear and explicit and this one fails to do so. If this standard refers simply to modeling of the four arithmetic operations, such modeling has already been done repeatedly in prior years' standards.
5. (3)(E): use prior knowledge of all four operations, including whole numbers and positive decimals, fractions, and mixed numbers not having fractions and decimals, within the same problem;
   I cannot parse this standard.
6. (4)(D): give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients;
   This seems confused. Division of two quantities is a quotient, so what is the meaning of the last clause?
7. (5)(A): represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions;
   How about this, instead:
   "*Use scale factors, tables, graphs and proportions to model abstract and real-world problems involving ratios and rates*".
   Many of the proposed standards should be written in this manner, with fewer gerunds and more clarity. The next point provides another example.
8. (7)(A): generate equivalent numerical expressions using order of operations, including positive exponents and prime factorization;
   Instead, I recommend the following:
   "*Simplify expressions, that include the four arithmetic operations and exponents, using order of operations and prime factorization*".

Grade 7

1. Standards (6), (7) and (8) carry the subject "proportionality." It should be "probability" instead.
2. Standards (10)(A), (10)(B) and (11)(A) ought to be merged. So should standards (10)(C) and (11)(B).
3. (11)(C) : determine the area of composite figures containing any combination of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles;
   Perhaps it would be better to remove the modifier "any," and add an "s" to "combination."
4. Standards (12)(A) and (13)(A) ought to be merged.

Grade 8

1. (2)(B): approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line;
   This standard needs to quantify the approximation and indicate how to derive the approximations.
2. (2)(C): convert between base-10 notation and scientific notation;
   Scientific notation is also a base-10 notation. Perhaps the intended meaning was to convert between standard decimal notation and scientific notation.
3. (5)(B): represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$;
   How can a linear situation be non-proportional?
4. (5)(E): solve problems involving direct variation;
   and
   (5)(F): solve directly proportional problems;
   These standards seem to duplicate each other.
5. (8)(D): write and solve equations using geometry concepts, including the angle relationships when parallel lines are cut by a transversal;
   Syntax is garbled. What is using "geometry concepts"? The writing or the solving? Or is it the problem that is geometrical in nature and it needs to be modeled and solved?
6. (8)(E) has the same syntactical issues as (8)(D): "write and solve equations using geometry concepts …"

Algebra I

1. (2)(E): write linear equations in two variables that contain a given point and are parallel to a given line;
   Is probably better written as:
   "write the equation of a line that contains a given point and is parallel to a given line;"
   Similar changes need to be made in (2)(F) and (2)(G)
2. (3)(B): calculate the rate of change of a linear function represented tabularly, graphically, and algebraically over a specified interval within mathematical and real-world problems;
   Is probably better written as:
   "calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems";
3. (12)(E): solve mathematic and scientific formulas, and other literal equations, for a specified variable.
   The standard needs to qualify the types of equations that it expects students to solve. As written, it can be anything from a simple linear equation to the Maxwell equations.
4. Algebra I content should include some amount of problem solving with quadratics.
5. Algebra I content would benefit from including some work on irrational numbers.

Algebra II

1. In standards (3) and (4), the Algebra II content would strongly benefit from solving those equations and systems of equations both out of context and in the context of problems.
2. (5)(D): solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations have real solutions;
   Probably garbled. The last clause is unclear.
3. (6)(B): solve cube root equations;
   Makes little sense. Perhaps more explicitness would help.
4. Mathematical induction is missing.

Geometry

1. (4): Logical argument and constructions. The student uses the process skills with inductive reasoning to understand geometric relationships.
   "Inductive reasoning" is probably a typo.
2. Geometry of polygons is missing.
3. Proof of the Pythagorean Theorem would strengthen the standards.
4. Exploring the area/volume relationship would strengthen the standards.

Precalculus

1. (5)(B): evaluate finite sums and geometric series, when possible, written in sigma notation;
   (5)(E): calculate the nth term of a geometric series, the nth partial sum of a geometric series, and sum of an infinite geometric series when it exists;
   Both standards deal with geometric series and seem duplicative. I would recommend that the standards should be merged or the difference clarified.
2. (5)(A): represent finite sums and infinite series using sigma notation;
   (5)(E): represent arithmetic series and geometric series using sigma notation;
   Both standards seem duplicative, unless the authors have in mind series beyond arithmetic and geometrical.